LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2018**

16/17PMT3MC01 – TOPOLOGY

Date: 25-10-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all the questions. Each question carries 20 marks.

I a)1) Let X be a metric space. Then prove that any arbitrary union of open sets is open.

OR

a)2) Let X be a metric space. Then prove that any finite intersection of open sets is open. (5)

b)1) If a convergent sequence in a metric space has infinitely many points then prove that its limit is a limit point of the set of points of the sequence.

b)2) State and prove Cantor's intersection theorem. (5+10)

OR

c)1) Let X be a metric space. Then prove the following results:

(i) any intersection of closed sets in X is closed.

(ii) any finite union of closed sets in X is closed.

c)2) State and prove Baire's theorem. (6+9)

II a)1) Define metrizable space and relative topology

OR

a)2) Topology is also called as Rubber Sheet Geometry. Explain? (5)

b)1) State and prove Lindelaef's theorem.

b)2) State and prove Heine-Borel theorem. (6+9)

OR

c) Prove that a topological space is compact if every subbasic open cover has a finite subcover. (15)

III a)1) Prove that every compact subspace of a Hausdorff space is closed.

OR

a)2) Prove that the product of any non-empty class of Hausdorff spaces is a Hausdorff space. (5)

b)1) State Urysohn lemma.

b)2) State and prove Tietze Extension theorem. (3+12)

OR

c)1) State and prove Urysohn Imbedding theorem. (15)

IV a)1) Prove that any continuous image of a connected space is connected.

OR

a)2) Prove that the range of a continuous real function defined on a connected space an interval. (5)

b)1) Prove that a subspace of a real line R is connected if and only if it is an interval. In particular show that

R is connected.

b)2) Prove that the product of any non-empty class of connected spaces is connected.

b)3) Prove that the spaces R^n and C^n are connected. (6+4+5)

OR

c)1) Let X be a compact Hausdorff space. Then prove that X is totally disconnected if and only if it has an open base whose sets are also closed.

c)2) Define Locally connected space and prove: Let X be a locally connected space. If Y is an open subspace of X, then each component of Y is open in X. In particular, each component of X is open.

(6+9)

V a)1) Prove that X_∞ is compact

OR

a)2) Prove that X_∞ is Hausdorff. (5)

b) State and prove Weierstrass approximation theorem. (15)

OR

c) Quoting the required results, state and prove Real Stone Weierstrass theorem. (15)

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